Before
You found what combinations of angles are possible in a triangle.

Now
You will find possible side lengths of a triangle.

Why?
So you can find possible distances, as in Ex. 39.

Key Vocabulary
• side opposite, p. 241
• inequality, p. 876

Example 1  Relate side length and angle measure

Draw an obtuse scalene triangle. Find the largest angle and longest side and mark them in red. Find the smallest angle and shortest side and mark them in blue. What do you notice?

Solution

The longest side and largest angle are opposite each other. The shortest side and smallest angle are opposite each other.

The relationships in Example 1 are true for all triangles as stated in the two theorems below. These relationships can help you to decide whether a particular arrangement of side lengths and angle measures in a triangle may be possible.

Avoid Errors
Be careful not to confuse the symbol \( \angle \) meaning angle with the symbol \(<\) meaning is less than. Notice that the bottom edge of the angle symbol is horizontal.

Theorems

Theorem 5.10
If one side of a triangle is longer than another side, then the angle opposite the longer side is larger than the angle opposite the shorter side.

Proof: p. 329

\[ AB > BC, \text{ so } m\angle C > m\angle A. \]

Theorem 5.11
If one angle of a triangle is larger than another angle, then the side opposite the larger angle is longer than the side opposite the smaller angle.

Proof: Ex. 24, p. 340

\[ m\angle A > m\angle C, \text{ so } BC > AB. \]
**Example 2** Standardized Test Practice

**STAGE PROP** You are constructing a stage prop that shows a large triangular mountain. The bottom edge of the mountain is about 27 feet long, the left slope is about 24 feet long, and the right slope is about 20 feet long. You are told that one of the angles is about 46° and one is about 59°. What is the angle measure of the peak of the mountain?

![Diagram of a triangular mountain with side lengths and angles labeled.]

**Solution**

Draw a diagram and label the side lengths. The peak angle is opposite the longest side so, by Theorem 5.10, the peak angle is the largest angle.

The angle measures sum to 180°, so the third angle measure is 180° − (46° + 59°) = 75°. You can now label the angle measures in your diagram.

- The greatest angle measure is 75°, so the correct answer is C. A  B  C  D

**Guided Practice** for Examples 1 and 2

1. List the sides of ΔRST in order from shortest to longest.

2. Another stage prop is a right triangle with sides that are 6, 8, and 10 feet long and angles of 90°, about 37°, and about 53°. Sketch and label a diagram with the shortest side on the bottom and the right angle at the left.

**Proof** Theorem 5.10

**Given** ▶ BC > AB

**Prove** ▶ m∠BAC > m∠C

Locate a point D on BC such that DB = BA. Then draw AD. In the isosceles triangle ΔABD, ∠1 ≅ ∠2.

Because m∠BAC = m∠1 + m∠3, it follows that m∠BAC > m∠1. Substituting m∠2 for m∠1 produces m∠BAC > m∠2.

By the Exterior Angle Theorem, m∠2 = m∠3 + m∠C, so it follows that m∠2 > m∠C (see Exercise 27, page 332). Finally, because m∠BAC > m∠2 and m∠2 > m∠C, you can conclude that m∠BAC > m∠C.
**THE TRIANGLE INEQUALITY** Not every group of three segments can be used to form a triangle. The lengths of the segments must fit a certain relationship. For example, three attempted triangle constructions for sides with given lengths are shown below. Only the first set of side lengths forms a triangle.

If you start with the longest side and attach the other two sides at its endpoints, you can see that the other two sides are not long enough to form a triangle in the second and third figures. This leads to the Triangle Inequality Theorem.

**Example 3** Find possible side lengths

**Algebra** A triangle has one side of length 12 and another of length 8. Describe the possible lengths of the third side.

**Solution**

Let \(x\) represent the length of the third side. Draw diagrams to help visualize the small and large values of \(x\). Then use the Triangle Inequality Theorem to write and solve inequalities.

- **Small values of \(x\)**
  
  \[ x + 8 > 12 \]
  \[ x > 4 \]

- **Large values of \(x\)**
  
  \[ 8 + 12 > x \]
  \[ 20 > x \]

The length of the third side must be greater than 4 and less than 20.
1. **VOCABULARY** Use the diagram at the right. For each angle, name the side that is opposite that angle.

2. **WRITING** How can you tell from the angle measures of a triangle which side of the triangle is the longest? the shortest?

3. **MEASURING** Use a ruler and protractor to draw the given type of triangle. Mark the largest angle and longest side in red and the smallest angle and shortest side in blue. What do you notice?

   - Acute scalene
   - Right scalene
   - Obtuse isosceles

4. **WRITING** List the sides and the angles in order from smallest to largest.

   6. Side lengths: 3 m, 7 m, and 9 m, with longest side on the bottom
   7. Side lengths: 37 ft, 35 ft, and 12 ft, with shortest side at the right
   8. Side lengths: 11 in., 13 in., and 14 in., with middle-length side at the left

5. **MULTIPLE CHOICE** In \(\triangle RST\), which is a possible side length for \(ST\)?

   - A 7
   - B 8
   - C 9
   - D Cannot be determined

6. **DRAWING TRIANGLES** Sketch and label the triangle described.

   - Side lengths: about 3 m, 7 m, and 9 m, with longest side on the bottom
   - Angle measures: 16°, 41°, and 123°, with smallest angle at the left
   - Side lengths: 37 ft, 35 ft, and 12 ft, with shortest side at the right
   - Angle measures: about 71°, about 19°, and 90°, with right angle at the top
   - Side lengths: 11 in., 13 in., and 14 in., with middle-length side at the left
   - Two angle measures: about 48° and 71°, with largest angle at the top

7. **IDENTIFYING POSSIBLE TRIANGLES** Is it possible to construct a triangle with the given side lengths? If not, explain why not.

   - 6, 7, 11
   - 3, 6, 9
   - 28, 34, 39
   - 35, 120, 125
20. ★ MULTIPLE CHOICE Which group of side lengths can be used to construct a triangle?
   A  3 yd, 4 ft, 5 yd
   B  3 yd, 5 ft, 8 ft
   C  11 in., 16 in., 27 in.
   D  2 ft, 11 in., 12 in.

POSSIBLE SIDE LENGTHS Describe the possible lengths of the third side of the triangle given the lengths of the other two sides.
21. 5 inches, 12 inches
22. 3 meters, 4 meters
23. 12 feet, 18 feet
24. 10 yards, 23 yards
25. 2 feet, 40 inches
26. 25 meters, 25 meters

27. EXTERIOR ANGLE INEQUALITY Another triangle inequality relationship is given by the Exterior Inequality Theorem. It states:
The measure of an exterior angle of a triangle is greater than the measure of either of the nonadjacent interior angles.
Use a relationship from Chapter 4 to explain how you know that \( m \angle 1 > m \angle A \) and \( m \angle 1 > m \angle B \) in \( \triangle ABC \) with exterior angle \( \angle 1 \).

ERROR ANALYSIS Use Theorems 5.10–5.12 and the theorem in Exercise 27 to explain why the diagram must be incorrect.
28.
29.

30. ★ SHORT RESPONSE Explain why the hypotenuse of a right triangle must always be longer than either leg.

ORDERING MEASURES Is it possible to build a triangle using the given side lengths? If so, order the angles measures of the triangle from least to greatest.
31. \( PQ = \sqrt{58}, QR = 2\sqrt{13}, PR = 5\sqrt{2} \)
32. \( ST = \sqrt{29}, TU = 2\sqrt{17}, SU = 13.9 \)

ALGEBRA Describe the possible values of \( x \).
33.
34.

35. USING SIDE LENGTHS Use the diagram at the right. Suppose \( XY \) bisects \( \angle WYZ \). List all six angles of \( \triangle XYZ \) and \( \triangle WXY \) in order from smallest to largest. Explain your reasoning.
36. CHALLENGE The perimeter of \( \triangle HGF \) must be between what two integers? Explain your reasoning.

= WORKED-OUT SOLUTIONS on p. WS1 ★ = STANDARDIZED TEST PRACTICE
37. **TRAY TABLE** In the tray table shown, \( PQ \equiv PR \) and \( QR < PQ \). Write two inequalities about the angles in \( \triangle PQR \). What other angle relationship do you know?  

38. **INDIRECT MEASUREMENT** You can estimate the width of the river at point \( A \) by taking several sightings to the tree across the river at point \( B \). The diagram shows the results for locations \( C \) and \( D \) along the riverbank. Using \( \triangle BCA \) and \( \triangle BDA \), what can you conclude about \( AB \), the width of the river at point \( A \)? What could you do if you wanted a closer estimate?  

39. **EXTENDED RESPONSE** You are planning a vacation to Montana. You want to visit the destinations shown in the map.  
   a. A brochure states that the distance between Granite Peak and Fort Peck Lake is 1080 kilometers. Explain how you know that this distance is a misprint.  
   b. Could the distance from Granite Peak to Fort Peck Lake be 40 kilometers? Explain.  
   c. Write two inequalities to represent the range of possible distances from Granite Peak to Fort Peck Lake.  
   d. What can you say about the distance between Granite Peak and Fort Peck Lake if you know that \( m \angle 2 < m \angle 1 \) and \( m \angle 2 < m \angle 3 \)?  

**FORMING TRIANGLES** In Exercises 40–43, you are given a 24 centimeter piece of string. You want to form a triangle out of the string so that the length of each side is a whole number. Draw figures accurately.  

40. Can you decide if three side lengths form a triangle without checking all three inequalities shown for Theorem 5.12? If so, describe your shortcut.  
41. Draw four possible isosceles triangles and label each side length. Tell whether each of the triangles you formed is acute, right, or obtuse.  
42. Draw three possible scalene triangles and label each side length. Try to form at least one scalene acute triangle and one scalene obtuse triangle.  
43. List three combinations of side lengths that will not produce triangles.
44. **SIGHTSEEING** You get off the Washington, D.C., subway system at the Smithsonian Metro station. First you visit the Museum of Natural History. Then you go to the Air and Space Museum. You record the distances you walk on your map as shown. Describe the range of possible distances you might have to walk to get back to the Smithsonian Metro station.

45. **SHORT RESPONSE** Your house is 2 miles from the library. The library is \( \frac{3}{4} \) mile from the grocery store. What do you know about the distance from your house to the grocery store? Explain. Include the special case when the three locations are all in a straight line.

46. **ISOSCELES TRIANGLES** For what combinations of angle measures in an isosceles triangle are the congruent sides shorter than the base of the triangle? longer than the base of the triangle?

47. **PROVING THEOREM 5.12** Prove the Triangle Inequality Theorem.

**GIVEN** \( \triangle ABC \)

**PROVE**

(1) \( AB + BC > AC \)
(2) \( AC + BC > AB \)
(3) \( AB + AC > BC \)

**Plan for Proof** One side, say \( BC \), is longer than or at least as long as each of the other sides. Then (1) and (2) are true. To prove (3), extend \( AC \) to \( D \) so that \( AB \parallel AD \) and use Theorem 5.11 to show that \( DC > BC \).

48. **CHALLENGE** Prove the following statements.

a. The length of any one median of a triangle is less than half the perimeter of the triangle.

b. The sum of the lengths of the three medians of a triangle is greater than half the perimeter of the triangle.

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**MIXED REVIEW**

In Exercises 49 and 50, write the if-then form, the converse, the inverse, and the contrapositive of the given statement. (p. 79)

49. A redwood is a large tree.

50. \( 5x - 2 = 18 \), because \( x = 4 \).

51. A triangle has vertices \( A(22, 21), B(0, 0), \) and \( C(22, 2) \). Graph \( \triangle ABC \) and classify it by its sides. Then determine if it is a right triangle. (p. 217)

Graph figure \( LMNP \) with vertices \( L(-4, 6), M(4, 8), N(2, 2), \) and \( P(-4, 0) \). Then draw its image after the transformation. (p. 272)

52. \( (x, y) \rightarrow (x + 3, y - 4) \)

53. \( (x, y) \rightarrow (x, -y) \)

54. \( (x, y) \rightarrow (-x, y) \)