Investigating Geometry **ACTIVITY** *Use before Lesson 7.3*

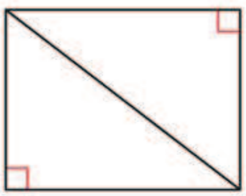
7.3 Similar Right Triangles

MATERIALS • rectangular piece of paper • ruler • scissors • colored pencils

QUESTION How are geometric means related to the altitude of a right triangle?

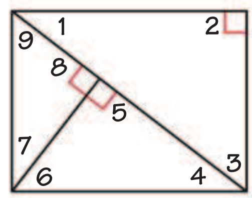
EXPLORE Compare right triangles

STEP 1



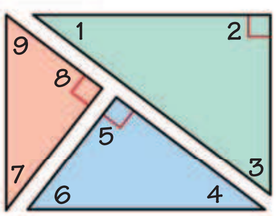
Draw a diagonal Draw a diagonal on your rectangular piece of paper to form two congruent right triangles.

STEP 2



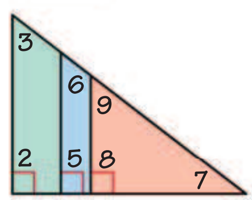
Draw an altitude Fold the paper to make an altitude to the hypotenuse of one of the triangles.

STEP 3



Cut and label triangles Cut the rectangle into the three right triangles that you drew. Label the angles and color the triangles as shown.

STEP 4



Arrange the triangles Arrange the triangles so ∠1, ∠4, and ∠7 are on top of each other as shown.

DRAW CONCLUSIONS Use your observations to complete these exercises

1. How are the two smaller right triangles related to the large triangle?
2. *Explain* how you would show that the green triangle is similar to the red triangle.
3. *Explain* how you would show that the red triangle is similar to the blue triangle.
4. The *geometric mean* of a and b is x if $\frac{a}{x} = \frac{x}{b}$. Write a proportion involving the side lengths of two of your triangles so that one side length is the geometric mean of the other two lengths in the proportion.

7.3 Use Similar Right Triangles

Before You identified the altitudes of a triangle.

Now You will use properties of the altitude of a right triangle.

Why? So you can determine the height of a wall, as in Example 4.

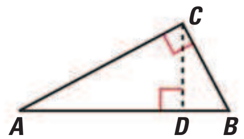
Key Vocabulary
- **altitude of a triangle,** *p. 320*
- **geometric mean,** *p. 359*
- **similar polygons,** *p. 372*

When the altitude is drawn to the hypotenuse of a right triangle, the two smaller triangles are similar to the original triangle and to each other.

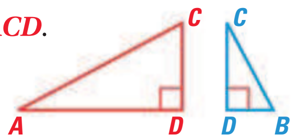
THEOREM *For Your Notebook*

THEOREM 7.5

If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.



$\triangle CBD \sim \triangle ABC$, $\triangle ACD \sim \triangle ABC$, and $\triangle CBD \sim \triangle ACD$.



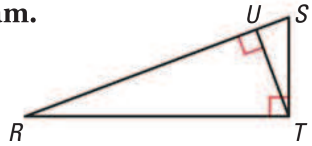
Proof: below; Ex. 35, p. 456

Plan for Proof of Theorem 7.5 First prove that $\triangle CBD \sim \triangle ABC$. Each triangle has a right angle and each triangle includes $\angle B$. The triangles are similar by the AA Similarity Postulate. Use similar reasoning to show that $\triangle ACD \sim \triangle ABC$.

To show $\angle CBD \sim \triangle ACD$, begin by showing $\angle ACD \cong \angle B$ because they are both complementary to $\angle DCB$. Each triangle also has a right angle, so you can use the AA Similarity Postulate.

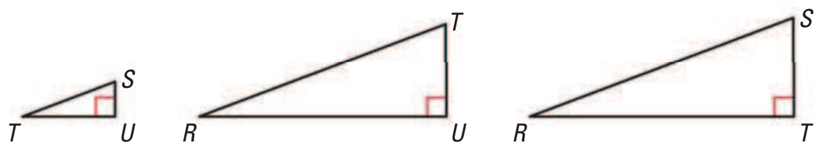
EXAMPLE 1 Identify similar triangles

Identify the similar triangles in the diagram.



Solution

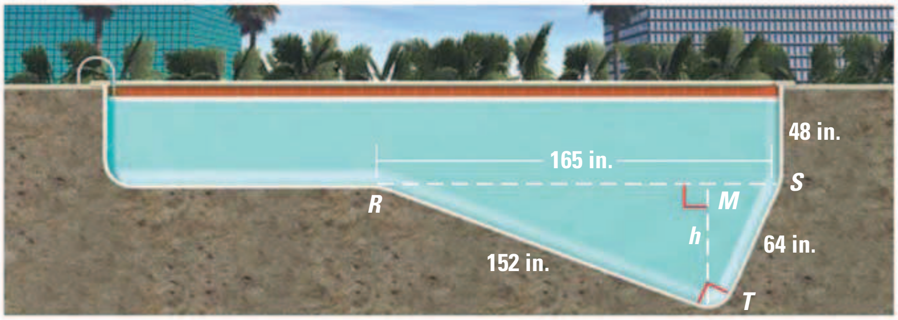
Sketch the three similar right triangles so that the corresponding angles and sides have the same orientation.



▶ $\triangle TSU \sim \triangle RTU \sim \triangle RST$

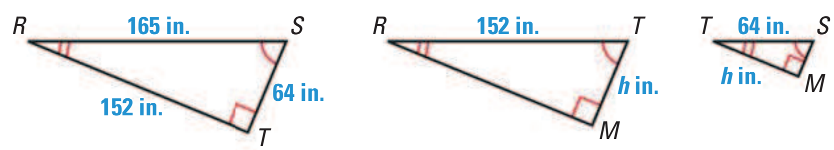
EXAMPLE 2 Find the length of the altitude to the hypotenuse

SWIMMING POOL The diagram below shows a cross-section of a swimming pool. What is the maximum depth of the pool?



Solution

STEP 1 **Identify** the similar triangles and sketch them.



$\triangle RST \sim \triangle RTM \sim \triangle TSM$

AVOID ERRORS
Notice that if you tried to write a proportion using $\triangle RTM$ and $\triangle TSM$, there would be two unknowns, so you would not be able to solve for *h*.

STEP 2 **Find** the value of h. Use the fact that $\triangle RST \sim \triangle RTM$ to write a proportion.

$\frac{TM}{ST} = \frac{TR}{SR}$ Corresponding side lengths of similar triangles are in proportion.

$\frac{h}{64} = \frac{152}{165}$ Substitute.

$165h = 64(152)$ Cross Products Property

$h \approx 59$ Solve for *h*.

STEP 3 **Read** the diagram above. You can see that the maximum depth of the pool is $h + 48$, which is about $59 + 48 = 107$ inches.

▶ The maximum depth of the pool is about 107 inches.

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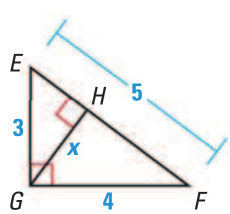
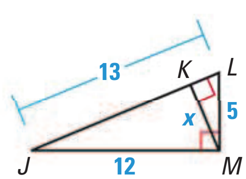
GUIDED PRACTICE for Examples 1 and 2

Identify the similar triangles. Then find the value of *x*.

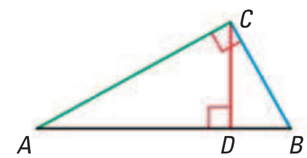
1.



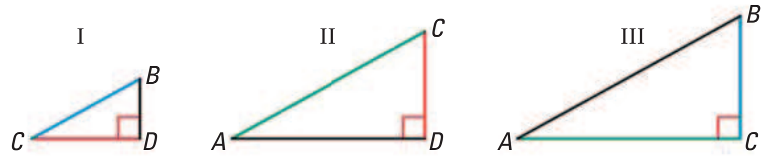
2.



GEOMETRIC MEANS In Lesson 6.1, you learned that the *geometric mean* of two numbers a and b is the positive number x such that $\frac{a}{x} = \frac{x}{b}$. Consider right $\triangle ABC$. From Theorem 7.5, you know that altitude $\overline{CD}$ forms two smaller triangles so that $\triangle CBD \sim \triangle ACD \sim \triangle ABC$.



READ SYMBOLS
Remember that an altitude is defined as a segment. So, $\overline{CD}$ refers to an altitude in $\triangle ABC$ and CD refers to its length.



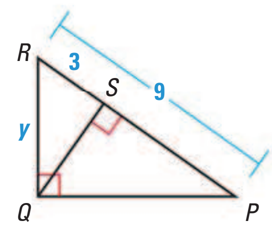
Notice that $\overline{CD}$ is the longer leg of $\triangle CBD$ and the shorter leg of $\triangle ACD$. When you write a proportion comparing the leg lengths of $\triangle CBD$ and $\triangle ACD$, you can see that CD is the geometric mean of BD and AD. As you see below, CB and AC are also geometric means of segment lengths in the diagram.

Proportions Involving Geometric Means in Right $\triangle ABC$

length of shorter leg of I / length of shorter leg of II →	$\frac{BD}{CD} = \frac{CD}{AD}$	← length of longer leg of I / length of longer leg of II
length of hypotenuse of III / length of hypotenuse of I →	$\frac{AB}{CB} = \frac{CB}{DB}$	← length of shorter leg of III / length of shorter leg of I
length of hypotenuse of III / length of hypotenuse of II →	$\frac{AB}{AC} = \frac{AC}{AD}$	← length of longer leg of III / length of longer leg of II

EXAMPLE 3 Use a geometric mean

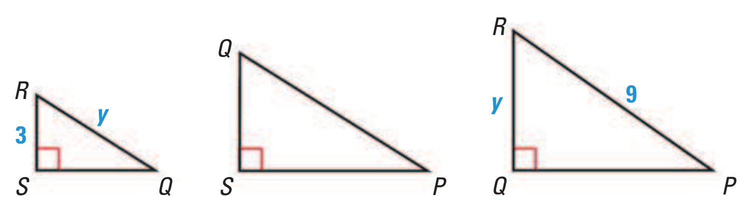
Find the value of y. Write your answer in simplest radical form.



Solution

REVIEW SIMILARITY
Notice that $\triangle RQS$ and $\triangle RPQ$ both contain the side with length y, so these are the similar pair of triangles to use to solve for y.

STEP 1 **Draw** the three similar triangles.



STEP 2 **Write** a proportion.

$$\frac{\text{length of hyp. of } \triangle RPQ}{\text{length of hyp. of } \triangle RQS} = \frac{\text{length of shorter leg of } \triangle RPQ}{\text{length of shorter leg of } \triangle RQS}$$

$\frac{9}{y} = \frac{y}{3}$ **Substitute.**

$27 = y^2$ **Cross Products Property**

$\sqrt{27} = y$ **Take the positive square root of each side.**

$3\sqrt{3} = y$ **Simplify.**

WRITE PROOFS
In Exercise 32 on page 455, you will use the geometric mean theorems to prove the Pythagorean Theorem.

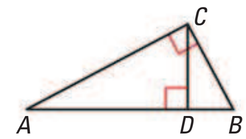
THEOREMS *For Your Notebook*

THEOREM 7.6 Geometric Mean (Altitude) Theorem

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

The length of the altitude is the geometric mean of the lengths of the two segments.

Proof: Ex. 36, p. 456



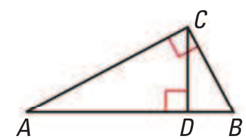
$$\frac{BD}{CD} = \frac{CD}{AD}$$

THEOREM 7.7 Geometric Mean (Leg) Theorem

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

The length of each leg of the right triangle is the geometric mean of the lengths of the hypotenuse and the segment of the hypotenuse that is adjacent to the leg.

Proof: Ex. 37, p. 456

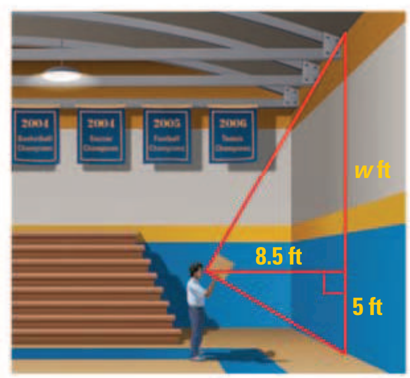


$$\frac{AB}{CB} = \frac{CB}{DB} \text{ and } \frac{AB}{AC} = \frac{AC}{AD}$$

EXAMPLE 4 Find a height using indirect measurement

ROCK CLIMBING WALL To find the cost of installing a rock wall in your school gymnasium, you need to find the height of the gym wall.

You use a cardboard square to line up the top and bottom of the gym wall. Your friend measures the vertical distance from the ground to your eye and the distance from you to the gym wall. Approximate the height of the gym wall.



Solution

By Theorem 7.6, you know that 8.5 is the geometric mean of w and 5.

$\frac{w}{8.5} = \frac{8.5}{5}$ **Write a proportion.**

$w \approx 14.5$ **Solve for *w*.**

▶ So, the height of the wall is $5 + w \approx 5 + 14.5 = 19.5$ feet.

✓ GUIDED PRACTICE for Examples 3 and 4

3. In Example 3, which theorem did you use to solve for y? *Explain*.

4. Mary is 5.5 feet tall. How far from the wall in Example 4 would she have to stand in order to measure its height?

7.3 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 15, and 29

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 19, 20, 31, and 34

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: Two triangles are ___?___ if their corresponding angles are congruent and their corresponding side lengths are proportional.

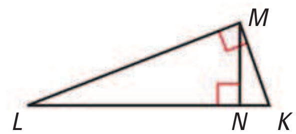
2. ★ **WRITING** In your own words, explain *geometric mean*.

EXAMPLE 1 on p. 449 for Exs. 3–4

IDENTIFYING SIMILAR TRIANGLES **Identify the three similar right triangles in the given diagram.**

3.

4.



EXAMPLE 2 on p. 450 for Exs. 5–7

FINDING ALTITUDES **Find the length of the altitude to the hypotenuse. Round decimal answers to the nearest tenth.**

5.

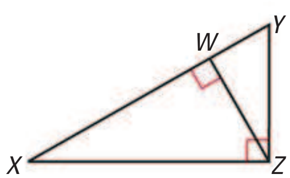
6.

7.

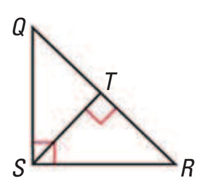
EXAMPLES 3 and 4 on pp. 451–452 for Exs. 8–18

COMPLETING PROPORTIONS **Write a similarity statement for the three similar triangles in the diagram. Then complete the proportion.**

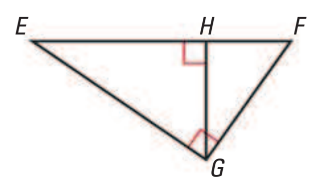
8. $\frac{XW}{?} = \frac{ZW}{YW}$



9. $\frac{?}{SQ} = \frac{SQ}{TQ}$

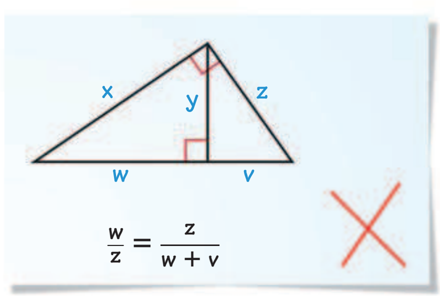


10. $\frac{EF}{EG} = \frac{EG}{?}$

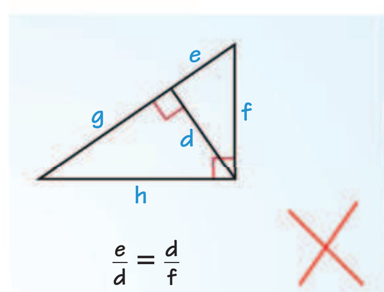


ERROR ANALYSIS ***Describe*** **and correct the error in writing a proportion for the given diagram.**

11.

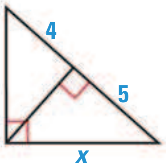


12.



FINDING LENGTHS **Find the value of the variable. Round decimal answers to the nearest tenth.**

13.



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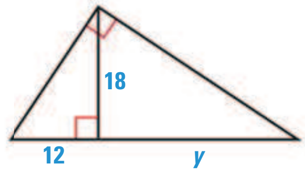
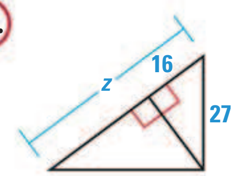


15.



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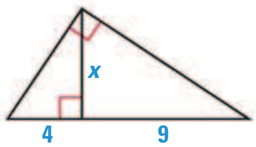

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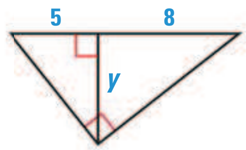
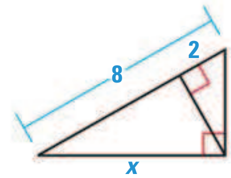


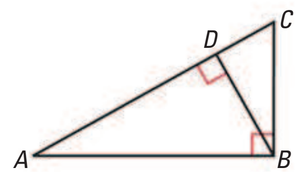
18.



19. ★ **MULTIPLE CHOICE** Use the diagram at the right. Decide which proportion is false.

Ⓐ $\frac{DB}{DC} = \frac{DA}{DB}$ Ⓑ $\frac{CA}{AB} = \frac{AB}{AD}$

Ⓒ $\frac{CA}{BA} = \frac{BA}{CA}$ Ⓓ $\frac{DC}{BC} = \frac{BC}{CA}$

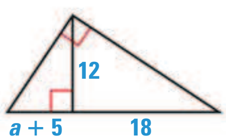


20. ★ **MULTIPLE CHOICE** In the diagram in Exercise 19 above, $AC = 36$ and $BC = 18$. Find AD. If necessary, round to the nearest tenth.

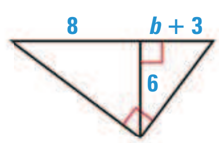
Ⓐ 9 Ⓑ 15.6 Ⓒ 27 Ⓓ 31.2

xy ALGEBRA **Find the value(s) of the variable(s).**

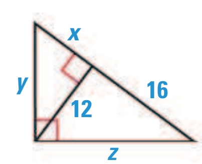
21.



22.

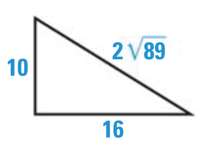


23.

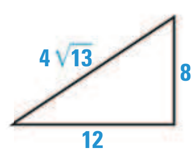


USING THEOREMS **Tell whether the triangle is a right triangle. If so, find the length of the altitude to the hypotenuse. Round decimal answers to the nearest tenth.**

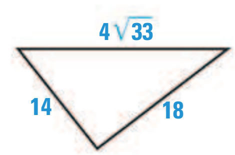
24.



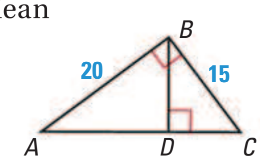
25.



26.



27. **FINDING LENGTHS** Use the Geometric Mean Theorems to find AC and BD.



28. **CHALLENGE** Draw a right isosceles triangle and label the two leg lengths x. Then draw the altitude to the hypotenuse and label its length y. Now draw the three similar triangles and label any side length that is equal to either x or y. What can you conclude about the relationship between the two smaller triangles? *Explain.*

◯ = **WORKED-OUT SOLUTIONS** on p. WS1 ★ = **STANDARDIZED TEST PRACTICE**

PROBLEM SOLVING

29. DOGHOUSE The peak of the doghouse shown forms a right angle. Use the given dimensions to find the height of the roof.

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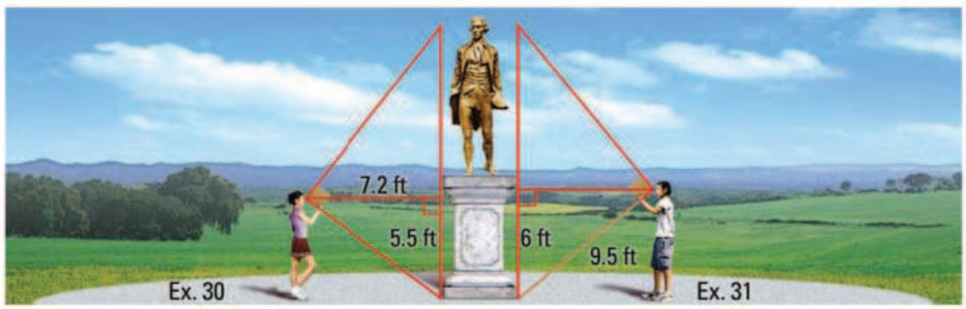
EXAMPLE 4 on p. 452 for Exs. 30–31

30. MONUMENT You want to determine the height of a monument at a local park. You use a cardboard square to line up the top and bottom of the monument. Mary measures the vertical distance from the ground to your eye and the distance from you to the monument. Approximate the height of the monument (as shown at the left below).



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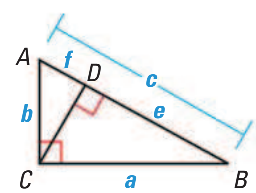
31. ★ SHORT RESPONSE Paul is standing on the other side of the monument in Exercise 30 (as shown at the right above). He has a piece of rope staked at the base of the monument. He extends the rope to the cardboard square he is holding lined up to the top and bottom of the monument. Use the information in the diagram above to approximate the height of the monument. Do you get the same answer as in Exercise 30? *Explain.*

32. PROVING THEOREM 7.1 Use the diagram of $\triangle ABC$. Copy and complete the proof of the Pythagorean Theorem.



GIVEN ▶ In $\triangle ABC$, $\angle BCA$ is a right angle.

PROVE ▶ $c^2 = a^2 + b^2$

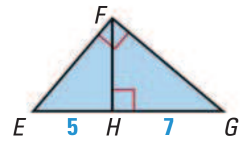
STATEMENTS	REASONS
1. Draw $\triangle ABC$. $\angle BCA$ is a right angle.	1. __?__
2. Draw a perpendicular from C to $\overline{AB}$.	2. Perpendicular Postulate
3. $\frac{c}{a} = \frac{a}{e}$ and $\frac{c}{b} = \frac{b}{f}$	3. __?__
4. $ce = a^2$ and $cf = b^2$	4. __?__
5. $ce + b^2 =$ __?__ $+ b^2$	5. Addition Property of Equality
6. $ce + cf = a^2 + b^2$	6. __?__
7. $c(e + f) = a^2 + b^2$	7. __?__
8. $e + f =$ __?__	8. Segment Addition Postulate
9. $c \cdot c = a^2 + b^2$	9. __?__
10. $c^2 = a^2 + b^2$	10. Simplify.

33. **MULTI-STEP PROBLEM** Use the diagram.

a. Name all the altitudes in $\triangle EGF$. *Explain.*

b. Find FH.

c. Find the area of the triangle.

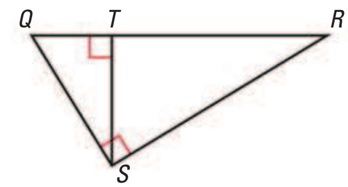


34. ★ **EXTENDED RESPONSE** Use the diagram.

a. Sketch the three similar triangles in the diagram. Label the vertices. *Explain* how you know which vertices correspond.

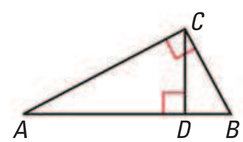
b. Write similarity statements for the three triangles.

c. Which segment's length is the geometric mean of RT and RQ? *Explain* your reasoning.



PROVING THEOREMS **In Exercises 35–37, use the diagram and GIVEN statements below.**

GIVEN ▶ $\triangle ABC$ is a right triangle.
Altitude $\overline{CD}$ is drawn to hypotenuse $\overline{AB}$.



35. Prove Theorem 7.5 by using the Plan for Proof on page 449.

36. Prove Theorem 7.6 by showing $\frac{BD}{CD} = \frac{CD}{AD}$.

37. Prove Theorem 7.7 by showing $\frac{AB}{CB} = \frac{CB}{DB}$ and $\frac{AB}{AC} = \frac{AC}{AD}$.

38. **CHALLENGE** The *harmonic mean* of a and b is $\frac{2ab}{a+b}$. The Greek mathematician Pythagoras found that three equally taut strings on stringed instruments will sound harmonious if the length of the middle string is equal to the harmonic mean of the lengths of the shortest and longest string.

a. Find the harmonic mean of 10 and 15.

b. Find the harmonic mean of 6 and 14.

c. Will equally taut strings whose lengths have the ratio 4 : 6 : 12 sound harmonious? *Explain* your reasoning.

MIXED REVIEW

PREVIEW
Prepare for Lesson 7.4 in Exs. 39–46.

Simplify the expression. *(p. 874)*

39. $\sqrt{27} \cdot \sqrt{2}$

40. $\sqrt{8} \cdot \sqrt{10}$

41. $\sqrt{12} \cdot \sqrt{7}$

42. $\sqrt{18} \cdot \sqrt{12}$

43. $\frac{5}{\sqrt{7}}$

44. $\frac{8}{\sqrt{11}}$

45. $\frac{15}{\sqrt{27}}$

46. $\frac{12}{\sqrt{24}}$

Tell whether the lines through the given points are *parallel*, *perpendicular*, or *neither*. *Justify* your answer. *(p. 171)*

47. **Line 1:** (2, 4), (4, 2)
Line 2: (3, 5), (−1, 1)

48. **Line 1:** (0, 2), (−1, −1)
Line 2: (3, 1), (1, −5)

49: **Line 1:** (1, 7), (4, 7)
Line 2: (5, 2), (7, 4)