10 Properties of Circles

Before

In previous chapters, you learned the following skills, which you'll use in Chapter 10: classifying triangles, finding angle measures, and solving equations.

Prerequisite Skills

VOCABULARY CHECK

Copy and complete the statement.

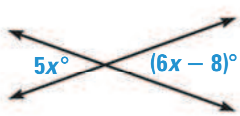
1. Two similar triangles have congruent corresponding angles and __?__ corresponding sides.
2. Two angles whose sides form two pairs of opposite rays are called __?__ .
3. The __?__ of an angle is all of the points between the sides of the angle.

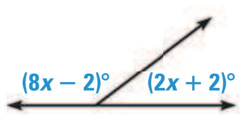
SKILLS AND ALGEBRA CHECK

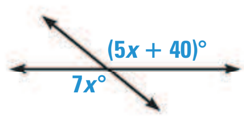
Use the Converse of the Pythagorean Theorem to classify the triangle. *(Review p. 441 for 10.1.)*

4. 0.6, 0.8, 0.9
5. 11, 12, 17
6. 1.5, 2, 2.5

Find the value of the variable. *(Review pp. 24, 35 for 10.2, 10.4.)*

7. 

8. 

9. 

@HomeTutor Prerequisite skills practice at classzone.com

Now

In Chapter 10, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 707. You will also use the key vocabulary listed below.

Big Ideas

1. **Using properties of segments that intersect circles**
2. **Applying angle relationships in circles**
3. **Using circles in the coordinate plane**

KEY VOCABULARY

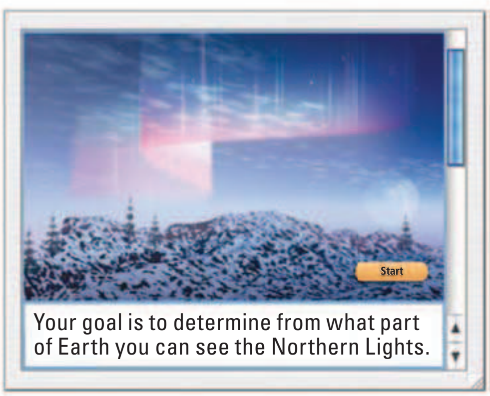
- circle, *p. 651*
  center, radius, diameter
- chord, *p. 651*
- secant, *p. 651*
- tangent, *p. 651*
- central angle, *p. 659*
- minor arc, *p. 659*
- major arc, *p. 659*
- semicircle, *p. 659*
- congruent circles, *p. 660*
- congruent arcs, *p. 660*
- inscribed angle, *p. 672*
- intercepted arc, *p. 672*
- standard equation of a circle, *p. 699*

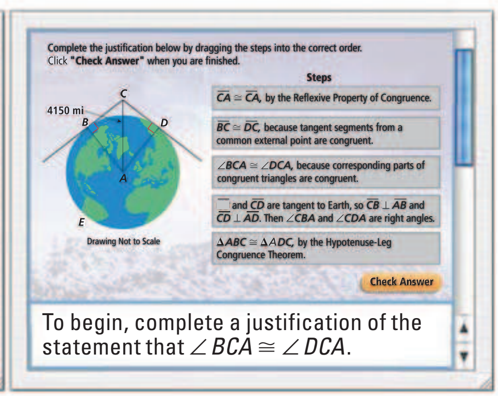
Why?

Circles can be used to model a wide variety of natural phenomena. You can use properties of circles to investigate the Northern Lights.

Animated Geometry

The animation illustrated below for Example 4 on page 682 helps you answer this question: From what part of Earth are the Northern Lights visible?





Animated Geometry at classzone.com

Other animations for Chapter 10: pages 655, 661, 671, 691, and 701

10.1 Explore Tangent Segments

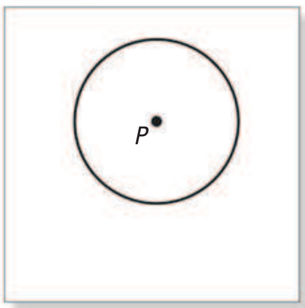
MATERIALS • compass • ruler

QUESTION How are the lengths of tangent segments related?

A line can intersect a circle at 0, 1, or 2 points. If a line is in the plane of a circle and intersects the circle at 1 point, the line is a *tangent*.

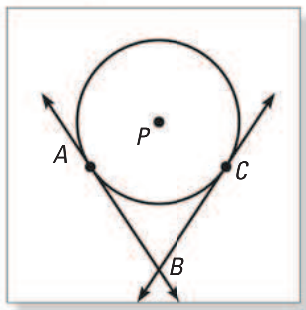
EXPLORE Draw tangents to a circle

STEP 1



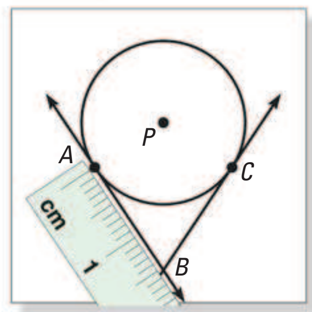
Draw a circle Use a compass to draw a circle. Label the center P.

STEP 2



Draw tangents Draw lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CB}$ so that they intersect $\odot P$ only at A and C, respectively. These lines are called *tangents*.

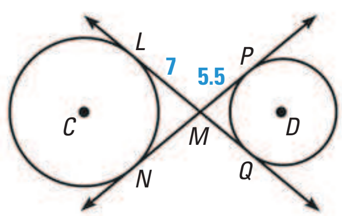
STEP 3



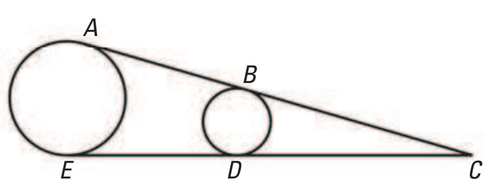
Measure segments $\overline{AB}$ and $\overline{CB}$ are called *tangent segments*. Measure and compare the lengths of the tangent segments.

DRAW CONCLUSIONS Use your observations to complete these exercises

1. Repeat Steps 1–3 with three different circles.
2. Use your results from Exercise 1 to make a conjecture about the lengths of tangent segments that have a common endpoint.
3. In the diagram, L, Q, N, and P are points of tangency. Use your conjecture from Exercise 2 to find LQ and NP if $LM = 7$ and $MP = 5.5$.



4. In the diagram below, A, B, D, and E are points of tangency. Use your conjecture from Exercise 2 to explain why $\overline{AB} \cong \overline{ED}$.



10.1 Use Properties of Tangents

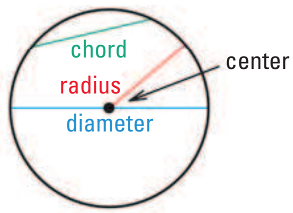
Before You found the circumference and area of circles.

Now You will use properties of a tangent to a circle.

Why? So you can find the range of a GPS satellite, as in Ex. 37.

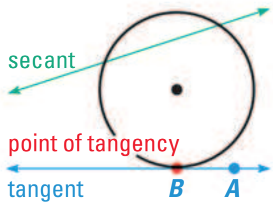
Key Vocabulary
- **circle**
  center, radius, diameter
- **chord**
- **secant**
- **tangent**

A **circle** is the set of all points in a plane that are equidistant from a given point called the **center** of the circle. A circle with center P is called "circle P" and can be written $\odot P$. A segment whose endpoints are the center and any point on the circle is a **radius**.



A **chord** is a segment whose endpoints are on a circle. A **diameter** is a chord that contains the center of the circle.

A **secant** is a line that intersects a circle in two points. A **tangent** is a line in the plane of a circle that intersects the circle in exactly one point, the *point of tangency*. The *tangent ray* $\overrightarrow{AB}$ and the *tangent segment* $\overline{AB}$ are also called tangents.

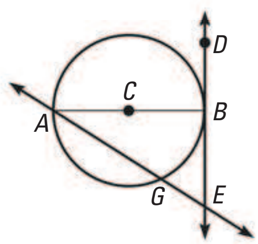


EXAMPLE 1 Identify special segments and lines

Tell whether the line, ray, or segment is best described as a *radius, chord, diameter, secant,* or *tangent* of $\odot C$.

a. $\overline{AC}$ **b.** $\overline{AB}$

c. $\overrightarrow{DE}$ **d.** $\overleftrightarrow{AE}$



Solution

a. $\overline{AC}$ is a radius because C is the center and A is a point on the circle.

b. $\overline{AB}$ is a diameter because it is a chord that contains the center C.

c. $\overrightarrow{DE}$ is a tangent ray because it is contained in a line that intersects the circle at only one point.

d. $\overleftrightarrow{AE}$ is a secant because it is a line that intersects the circle in two points.

✓ GUIDED PRACTICE for Example 1

1. In Example 1, what word best describes $\overline{AG}$? $\overline{CB}$?

2. In Example 1, name a tangent and a tangent segment.

READ VOCABULARY
The plural of radius is *radii*. All radii of a circle are congruent.

RADIUS AND DIAMETER The words *radius* and *diameter* are used for lengths as well as segments. For a given circle, think of *a radius* and *a diameter* as segments and *the radius* and *the diameter* as lengths.

EXAMPLE 2 Find lengths in circles in a coordinate plane

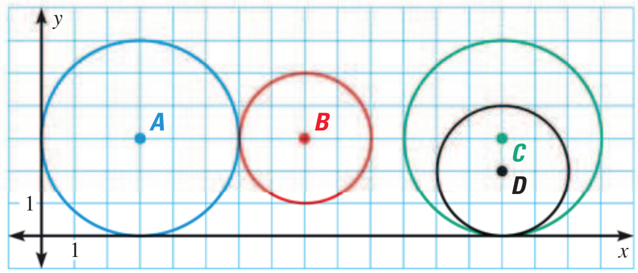
Use the diagram to find the given lengths.

a. Radius of $\odot A$

b. Diameter of $\odot A$

c. Radius of $\odot B$

d. Diameter of $\odot B$



Solution

a. The radius of $\odot A$ is 3 units.

b. The diameter of $\odot A$ is 6 units.

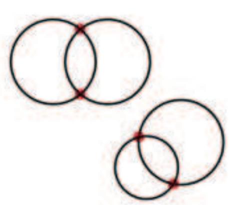
c. The radius of $\odot B$ is 2 units.

d. The diameter of $\odot B$ is 4 units.

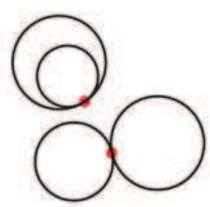
✓ GUIDED PRACTICE for Example 2

3. Use the diagram in Example 2 to find the radius and diameter of $\odot C$ and $\odot D$.

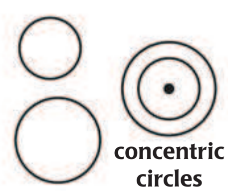
COPLANAR CIRCLES Two circles can intersect in two points, one point, or no points. Coplanar circles that intersect in one point are called *tangent circles*. Coplanar circles that have a common center are called *concentric*.



2 points of intersection



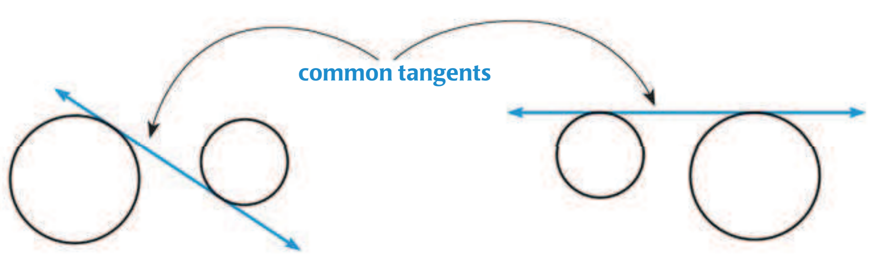
1 point of intersection (tangent circles)



no points of intersection

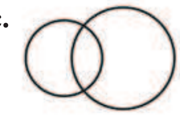
READ VOCABULARY
A line that intersects a circle in exactly one point is said to be *tangent* to the circle.

COMMON TANGENTS A line, ray, or segment that is tangent to two coplanar circles is called a *common tangent*.



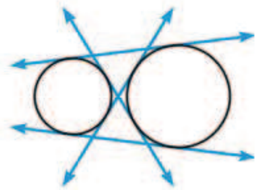
EXAMPLE 3 Draw common tangents

Tell how many common tangents the circles have and draw them.

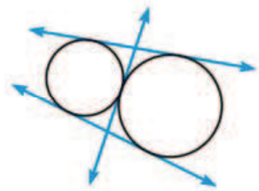
a. 
b. 
c. 

Solution

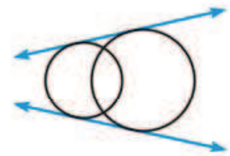
a. 4 common tangents



b. 3 common tangents

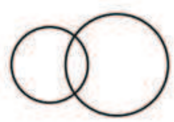


c. 2 common tangents



✓ GUIDED PRACTICE for Example 3

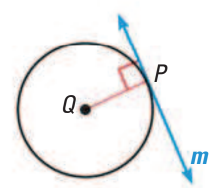
Tell how many common tangents the circles have and draw them.

4. 
5. 
6. 

THEOREM *For Your Notebook*

THEOREM 10.1

In a plane, a line is tangent to a circle if and only if the line is perpendicular to a radius of the circle at its endpoint on the circle.

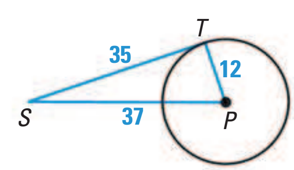


Line m is tangent to $\odot Q$ if and only if $m \perp \overline{QP}$.

Proof: Exs. 39–40, p. 658

EXAMPLE 4 Verify a tangent to a circle

In the diagram, $\overline{PT}$ is a radius of $\odot P$. Is $\overline{ST}$ tangent to $\odot P$?

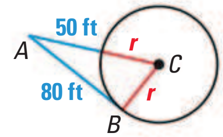


Solution

Use the Converse of the Pythagorean Theorem. Because $12^2 + 35^2 = 37^2$, $\triangle PST$ is a right triangle and $\overline{ST} \perp \overline{PT}$. So, $\overline{ST}$ is perpendicular to a radius of $\odot P$ at its endpoint on $\odot P$. By Theorem 10.1, $\overline{ST}$ is tangent to $\odot P$.

EXAMPLE 5 Find the radius of a circle

In the diagram, B is a point of tangency. Find the radius r of $\odot C$.



Solution

You know from Theorem 10.1 that $\overline{AB} \perp \overline{BC}$, so $\triangle ABC$ is a right triangle. You can use the Pythagorean Theorem.

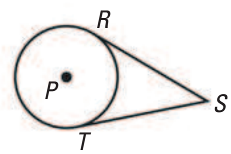
$AC^2 = BC^2 + AB^2$	**Pythagorean Theorem**
$(r + 50)^2 = r^2 + 80^2$	**Substitute.**
$r^2 + 100r + 2500 = r^2 + 6400$	**Multiply.**
$100r = 3900$	**Subtract from each side.**
$r = 39$ ft	**Divide each side by 100.**

THEOREM *For Your Notebook*

THEOREM 10.2

Tangent segments from a common external point are congruent.

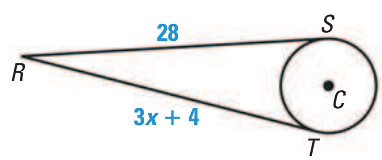
Proof: Ex. 41, p. 658



If $\overline{SR}$ and $\overline{ST}$ are tangent segments, then $\overline{SR} \cong \overline{ST}$.

EXAMPLE 6 Find the radius of a circle

$\overline{RS}$ is tangent to $\odot C$ at S and $\overline{RT}$ is tangent to $\odot C$ at T. Find the value of x.

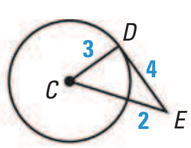


Solution

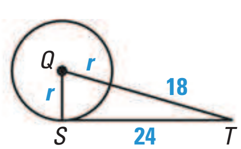
$RS = RT$	**Tangent segments from the same point are $\cong$.**
$28 = 3x + 4$	**Substitute.**
$8 = x$	**Solve for x.**

GUIDED PRACTICE for Examples 4, 5, and 6

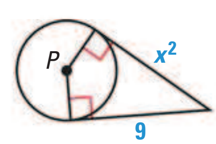
7. Is $\overline{DE}$ tangent to $\odot C$?



8. $\overline{ST}$ is tangent to $\odot Q$. Find the value of r.



9. Find the value(s) of x.



10.1 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 19, and 37

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 29, 33, and 38

SKILL PRACTICE

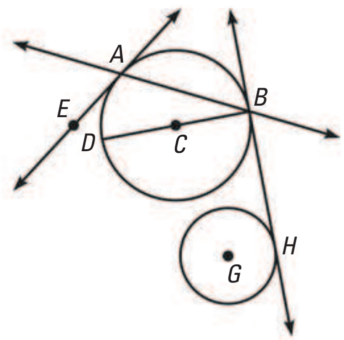
1. **VOCABULARY** Copy and complete: The points A and B are on $\odot C$. If C is a point on $\overline{AB}$, then $\overline{AB}$ is a __?__.

2. ★ **WRITING** Explain how you can determine from the context whether the words *radius* and *diameter* are referring to a segment or a length.

EXAMPLE 1 on p. 651 for Exs. 3–11

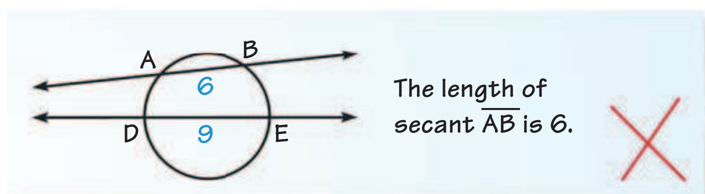
MATCHING TERMS **Match the notation with the term that best describes it.**

3. B	A. Center
4. $\overleftrightarrow{BH}$	B. Radius
5. $\overline{AB}$	C. Chord
6. $\overleftrightarrow{AB}$	D. Diameter
7. $\overleftrightarrow{AE}$	E. Secant
8. G	F. Tangent
9. $\overline{CD}$	G. Point of tangency
10. $\overline{BD}$	H. Common tangent



AnimatedGeometry at classzone.com

11. **ERROR ANALYSIS** *Describe* and correct the error in the statement about the diagram.



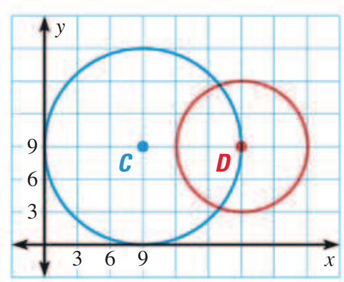
EXAMPLES 2 and 3 on pp. 652–653 for Exs. 12–17

COORDINATE GEOMETRY **Use the diagram at the right.**

12. What are the radius and diameter of $\odot C$?

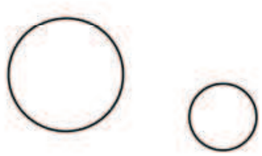
13. What are the radius and diameter of $\odot D$?

14. Copy the circles. Then draw all the common tangents of the two circles.

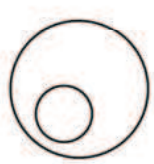


DRAWING TANGENTS **Copy the diagram. Tell how many common tangents the circles have and draw them.**

15.



16.

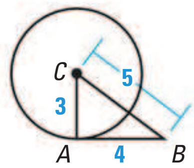


17.

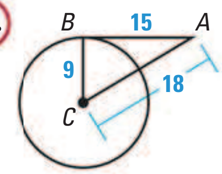
EXAMPLE 4
on p. 653
for Exs. 18–20

DETERMINING TANGENCY Determine whether $\overline{AB}$ is tangent to $\odot C$. *Explain.*

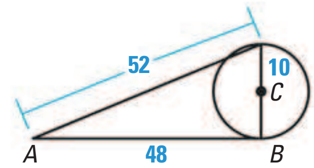
18.



19.



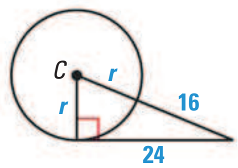
20.



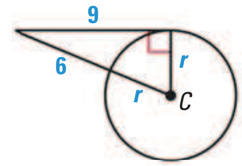
EXAMPLES 5 and 6
on p. 654
for Exs. 21–26

ALGEBRA Find the value(s) of the variable. In Exercises 24–26, *B* and *D* are points of tangency.

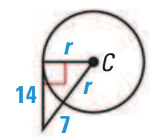
21.



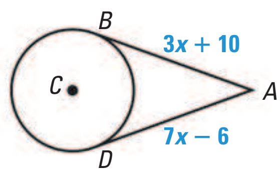
22.



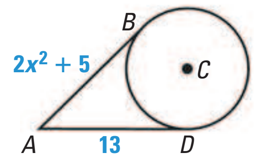
23.



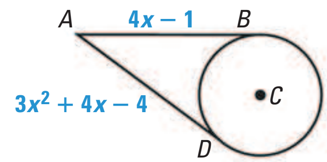
24.



25.

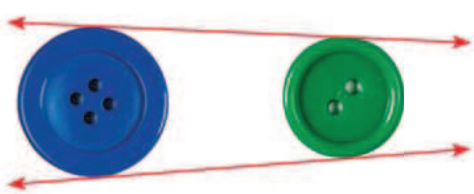


26.

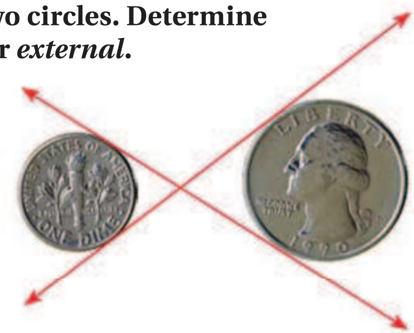


COMMON TANGENTS A *common internal tangent* intersects the segment that joins the centers of two circles. A *common external* tangent does not intersect the segment that joins the centers of the two circles. Determine whether the common tangents shown are *internal* or *external*.

27.



28.



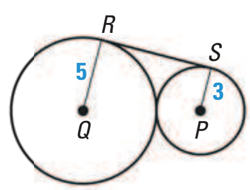
29. ★ **MULTIPLE CHOICE** In the diagram, $\odot P$ and $\odot Q$ are tangent circles. $\overline{RS}$ is a common tangent. Find RS.

Ⓐ $-2\sqrt{15}$

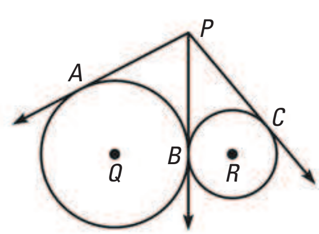
Ⓑ 4

Ⓒ $2\sqrt{15}$

Ⓓ 8



30. REASONING In the diagram, $\overrightarrow{PB}$ is tangent to $\odot Q$ and $\odot R$. *Explain* why $\overline{PA} \cong \overline{PB} \cong \overline{PC}$ even though the radius of $\odot Q$ is not equal to the radius of $\odot R$.

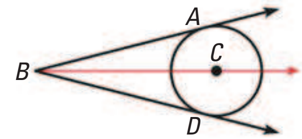


31. TANGENT LINES When will two lines tangent to the same circle not intersect? Use Theorem 10.1 to *explain* your answer.

○ = **WORKED-OUT SOLUTIONS** on p. WS1

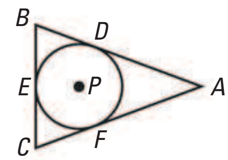
★ = **STANDARDIZED TEST PRACTICE**

32. **ANGLE BISECTOR** In the diagram at right, A and D are points of tangency on $\odot C$. *Explain* how you know that $\overrightarrow{BC}$ bisects $\angle ABD$. (*Hint*: Use Theorem 5.6, page 310.)



33. ★ **SHORT RESPONSE** For any point outside of a circle, is there ever only one tangent to the circle that passes through the point? Are there ever more than two such tangents? *Explain* your reasoning.

34. **CHALLENGE** In the diagram at the right, $AB = AC = 12$, $BC = 8$, and all three segments are tangent to $\odot P$. What is the radius of $\odot P$?



PROBLEM SOLVING

BICYCLES On modern bicycles, rear wheels usually have *tangential spokes*. Occasionally, front wheels have *radial spokes*. Use the definitions of *tangent* and *radius* to determine if the wheel shown has *tangential spokes* or *radial spokes*.

35.

36.

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EXAMPLE 4
on p. 653
for Ex. 37

37. **GLOBAL POSITIONING SYSTEM (GPS)** GPS satellites orbit about 11,000 miles above Earth. The mean radius of Earth is about 3959 miles. Because GPS signals cannot travel through Earth, a satellite can transmit signals only as far as points A and C from point B, as shown. Find BA and BC to the nearest mile.

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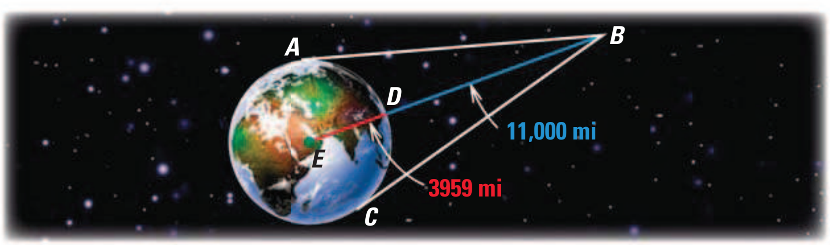
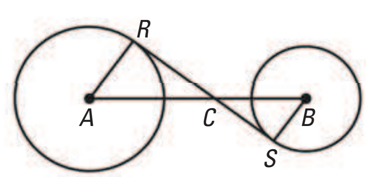
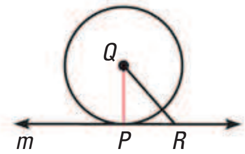


38. ★ **SHORT RESPONSE** In the diagram, $\overline{RS}$ is a common internal tangent (see Exercises 27–28) to $\odot A$ and $\odot B$. Use similar triangles to *explain* why $\frac{AC}{BC} = \frac{RC}{SC}$.



39. **PROVING THEOREM 10.1** Use parts (a)–(c) to prove indirectly that if a line is tangent to a circle, then it is perpendicular to a radius.



GIVEN ▶ Line m is tangent to $\odot Q$ at P.

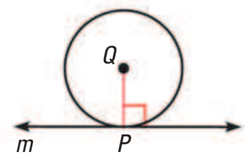
PROVE ▶ $m \perp \overline{QP}$

a. Assume m is not perpendicular to $\overline{QP}$. Then the perpendicular segment from Q to m intersects m at some other point R. Because m is a tangent, R cannot be inside $\odot Q$. *Compare* the length QR to QP.

b. Because $\overline{QR}$ is the perpendicular segment from Q to m, $\overline{QR}$ is the shortest segment from Q to m. Now *compare* QR to QP.

c. Use your results from parts (a) and (b) to complete the indirect proof.

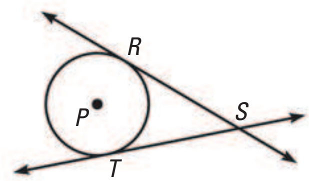
40. **PROVING THEOREM 10.1** Write an indirect proof that if a line is perpendicular to a radius at its endpoint, the line is a tangent.



GIVEN ▶ $m \perp \overline{QP}$

PROVE ▶ Line m is tangent to $\odot Q$.

41. **PROVING THEOREM 10.2** Write a proof that tangent segments from a common external point are congruent.

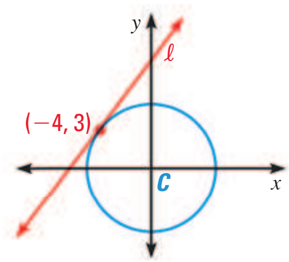


GIVEN ▶ $\overline{SR}$ and $\overline{ST}$ are tangent to $\odot P$.

PROVE ▶ $\overline{SR} \cong \overline{ST}$

Plan for Proof Use the Hypotenuse–Leg Congruence Theorem to show that $\triangle SRP \cong \triangle STP$.

42. **CHALLENGE** Point C is located at the origin. Line ℓ is tangent to $\odot C$ at $(-4, 3)$. Use the diagram at the right to complete the problem.



a. Find the slope of line ℓ.

b. Write the equation for ℓ.

c. Find the radius of $\odot C$.

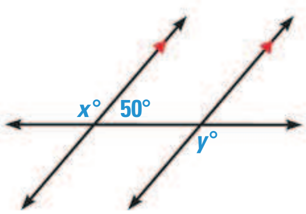
d. Find the distance from ℓ to $\odot C$ along the y-axis.

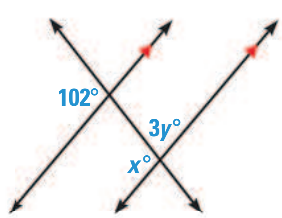
MIXED REVIEW

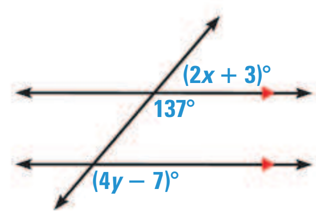
PREVIEW
Prepare for Lesson 10.2 in Ex. 43.

43. D is in the interior of $\angle ABC$. If $m\angle ABD = 25°$ and $m\angle ABC = 70°$, find $m\angle DBC$. *(p. 24)*

Find the values of x and y. *(p. 154)*

44. 

45. 

46. 

47. A triangle has sides of lengths 8 and 13. Use an inequality to describe the possible length of the third side. What if two sides have lengths 4 and 11? *(p. 328)*